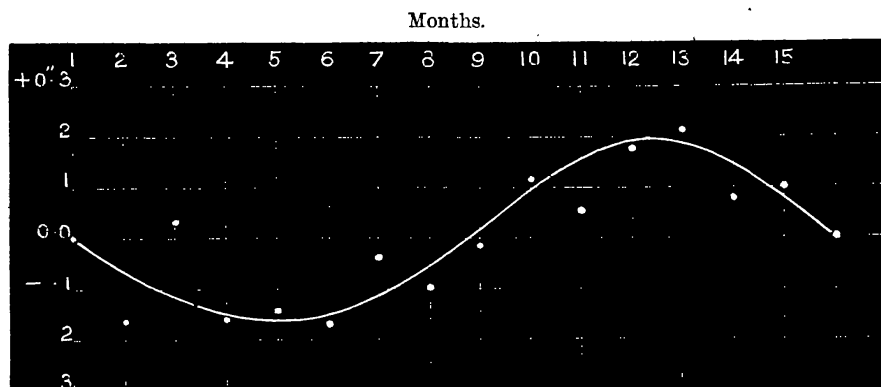


is more or less removed by the application of the correction for an assumed monthly difference in the R—D correction, and



2nd Supposition (corrected).

further evidence is needed to confirm this. The investigation will be continued, and the present paper must be regarded as preliminary merely; but the matter is of sufficient importance to make it advisable to publish these results at once.

*On the Systematic Errors of the Moon in Right Ascension.*

By H. H. Turner, M.A., B.Sc.

In February 1892 Captain Grant, of the Ordnance Survey, consulted the Astronomer Royal concerning operations for fixing the boundary of Mashonaland, in concert with Portuguese officers, by parallels of latitude and longitude.

It was decided that he should determine the requisite longitudes by Moon-culminations, taking with him a portable transit by Troughton & Simms. Application has several times been made to the Astronomer Royal, in connection with similar operations, for a list of the Moon's errors on certain dates, as determined by the Greenwich Transit Circle, but in the present instance the actual landmarks were to be fixed on the spot, and hence retrospective corrections depending on information supplied from Greenwich were out of the question. It remained to be decided whether from recent observations any prospective corrections to the Moon's place could be tabulated and furnished to Captain Grant before his departure. For this purpose a complete list of observed errors of the Moon in R.A. for the years 1883–1891 was formed. The year 1883 was the first in which Newcomb's corrections to Hansen's tables were applied in the *Nautical Almanac*; and quite apart from any theoretical question as to the exact relation of these corrections to Hansen's tables, their adoption in the *Nautical Almanac* has had the advantage of bringing readily into prominence several features of the residuals previously masked by the large corrections which had been

required by those tables. As an example, I may quote the annual mean error in R.A. of the Moon, given in the annual reports of the Astronomer Royal to the Society, or to the Board of Visitors. The latter value, computed in May, which sometimes differs slightly from the former, computed in February, is to be preferred; for by May the observations of the previous year have been examined more thoroughly, and numerical errors, if any, corrected. The following is a list of such annual means:—

	Report to			Report to	
	R.A.S. s	B. of V. s		R.A.S. s	B. of V. s
1883	+ 0.032	+ 0.031	1888	+ 0.079	+ 0.090
1884	+ 0.021	+ 0.018	1889	+ 0.010	+ 0.010
1885	+ 0.028	+ 0.024	1890	+ 0.020	+ 0.020
1886	+ 0.029	+ 0.029	1891	+ 0.079	+ 0.077
1887	+ 0.068	+ 0.059	Mean	+ 0.041	+ 0.040

The persistently positive sign of this quantity is noteworthy; and it is quite possible that the departures from the mean value may be accidental.

Again, there is a persistent difference, as will appear presently, between the errors before and after Full Moon, which may apparently be predicted empirically without making any attempt to refer it to one of several possible causes.

The aforesaid errors for 1883–1891 were arranged so that corresponding lunations in each year (the correspondence being, of course, only approximate) and corresponding days in each lunation (reckoning from Full Moon) were taken together. There were naturally many gaps in each lunation—days on which no meridian observation had been obtained at Greenwich—but from the data available an attempt was made to estimate the mean error on five dates in each lunation—viz. nine days before Full Moon, five days before, Full Moon, five days after, and nine days after. This was done for the mean January lunation, the mean February lunation, &c.; that is all the lunations where the Full Moon fell in January were considered “corresponding” and taken together, as remarked above; and if there had been an observation on every day, we should have had forty-five errors of which to take the mean to obtain the mean error at Full Moon in January—viz. a set of five days in each year comprising Full Moon and two days each side; and similarly for the other epochs. As a matter of fact, instead of forty-five errors there were generally only about fifteen, and hence came considerable uncertainty in estimating correctly.

The first point noticed from the results of this work was that the errors after the Full were algebraically greater than those

before. Further, that this difference was more marked in the summer than in the winter months. By a graphical process the following mean differences were deduced between the errors before and after Full, a positive sign indicating that the error after the Full is greater than that before:—

Jan.	<sup>s</sup> +0.04	July	<sup>s</sup> +0.17
Feb.	+0.11	Aug.	+0.13
Mar.	+0.16	Sept.	+0.07
Apr.	+0.20	Oct.	+0.01
May	+0.21	Nov.	−0.02
June	+0.20	Dec.	+0.02

The change at Full has been treated for simplicity as a discontinuity and not as a progressive change. The whole process being, in fact, of a tentative character, it is not advisable to give figures in greater detail; but it may be remarked that the adoption of an erroneous semi-diameter is a possible explanation of this phenomenon; for errors after the Full are deduced from observations of the second limb only, and before from those of the first. Now, if the adopted semi-diameter is too large, too large a quantity will be subtracted from transits after the Full; the observed "transit of centre" will be too small, and the error of the tables consequently a positive quantity; while before the Full the reverse is the case. Again, the adopted semi-diameter may be large owing to the spurious border, due to irradiation, which surrounds the limb of the Moon, and has been recognised as explaining "projection" in occultations of bright stars, and which would be most emphatic at midnight, the time of transit of Full Moon, when the diameter is usually measured with meridian instruments; whereas at the transits of First and Last Quarter Moons the irradiation border would be lost in the twilight background; and this would be especially the case in the summer. If this explanation be the true one, then the use of errors obtained by a large instrument for correction of those with a small is of somewhat doubtful expediency.

Next it was noticed that, considering the mean errors on any given date (nine days before Full, five days before, &c.) in, say, a January lunation, there was a certain progression from year to year; and it was hoped that by plotting these errors as ordinates against the years as abscissæ, a curve might be obtained sufficiently definite to allow of extension for one year, and hence the approximate error on this date for January 1893, &c. might be predicted. The great drawback to the method is the number of gaps in the different series; and as a first attempt two or three years were taken together to remedy this defect. The

resulting predictions for January and February 1893 were compared with the actual observations which were already available, and it was seen that no great measure of success had been attained. A second attempt was then made by first filling up the gaps by estimation from neighbouring dates, and the results of this attempt were more satisfactory, and were adopted for use.

Since the subjoined table was furnished to Captain Grant, the Moon has now been observed throughout the period under consideration, and it may, therefore, interest the Fellows of the Society to note the measure of success attained by a purely empirical method in a case of this kind. From what has been said above, it will be clear that this note is in no way intended as a serious contribution to lunar theory.

It will be seen that predictions were not made for the earlier days in each lunation, though the observations with the Transit Circle are printed.

*Corrections to R.A. of Moon in 1892. Predicted approximately from the errors 1883—1891 in Transit Circle Observations at Royal Observatory, Greenwich.*

	Pred.	Obs.	O—P
	<sup>s</sup>	<sup>s</sup>	<sup>s</sup>
May 3	+0.14	...	...
4	+0.12	...	...
5	+0.11	...	...
6	+0.09	+0.15	+0.06
7	+0.08	...	...
8	+0.07	−0.05	−0.12
9	+0.05	+0.16	+0.11
10	+0.03	+0.10	+0.07
11	0.00	−0.09	−0.09
12	−0.04	−0.10	−0.06
13	−0.10	...	...
14	−0.15	−0.02	+0.13
15	−0.19	−0.20	−0.01
16	−0.23	...	...
17	−0.27	...	...
18	−0.30	−0.13	+0.17
Numerical Sum ...		1.00	0.82

Nov. 1892.

*Moon in Right Ascension.*

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	Prel.	Obs.	O-P
	<sup>s</sup>	<sup>s</sup>	<sup>s</sup>
May 30	...	-0.08	...
June 1	...	+0.07	...
2	...	+0.07	...
3	+0.19	...	...
4	+0.16	...	...
5	+0.09	-0.01	-0.10
6	+0.02	...	...
7	-0.05	-0.04	+0.01
8	-0.08	-0.11	-0.03
9	-0.10	-0.06	+0.04
10	-0.13	-0.02	+0.11
11	-0.17	...	...
12	-0.21	-0.25	-0.04
13	-0.25	...	...
14	-0.28	...	...
15	-0.31	...	...
16	-0.33	...	...
17	-0.35	-0.40	-0.05
Numerical Sum	...	0.89	0.38

	Pred.	Obs.	O-P
	<sup>s</sup>	<sup>s</sup>	<sup>s</sup>
July 2	+0.12	+0.11	-0.01
3	+0.07	-0.04	-0.11
4	+0.01	+0.02	+0.01
5	-0.04	...	...
6	-0.09	...	...
7	-0.12	+0.09	+0.21
8	-0.13	-0.03	+0.10
9	-0.15	-0.21	-0.06
10	-0.19	-0.36	-0.17
11	-0.24	...	...
12	-0.28	...	...
13	-0.29	...	...
14	-0.29	...	...
15	-0.30	...	...
16	-0.30	...	...
Numerical Sum	...	0.86	0.67

	Pred.	Obs.	O-P
	s	s	s
July 28	...	+0.14	...
29	...	+0.10	...
30	...	+0.11	...
31	+0.10	...	...
Aug. 1	+0.06	...	...
2	+0.01	...	...
3	-0.04	...	...
4	-0.08	+0.17	+0.25
5	-0.11	...	...
6	-0.13	+0.11	+0.24
7	-0.15	...	...
8	-0.15	...	...
9	-0.16	...	...
10	-0.16	-0.52	-0.36
11	-0.17	-0.40	-0.23
12	-0.17	-0.41	-0.24
13	-0.18	-0.49	-0.31
14	-0.18	-0.46	-0.28
15	-0.17	-0.42	-0.25
16	...	-0.39	...
Numerical Sum ...		2.98	2.16
	Pred.	Obs.	O-P
	s	s	s
Aug. 26	+	+0.17	...
27	...	...	...
28	...	+0.14	...
29	...	...	...
30	+0.16	+0.21	+0.05
31	+0.16	...	...
Sept. 1	+0.14	-0.06	-0.20
2	+0.12	...	...
3	+0.09	+0.16	+0.07
4	+0.04	-0.13	-0.17
5	-0.02	-0.19	-0.17
6	-0.06	-0.22	-0.16
7	-0.10	-0.26	-0.16
8	-0.13	-0.37	-0.24
9	-0.16	...	...
10	-0.16	-0.38	-0.22
11	-0.15	-0.38	-0.23
12	-0.14	...	...
13	...	-0.45	...
Numerical Sum ...		2.36	1.07

*Corrections to Hansen's Tables de la Lune.* By James Gordon.

(Communicated by the Rev. S. J. Johnson, M.A.)

I have reviewed Professor Newcomb's "Researches on the Motion of the Moon," part 1, and I have accepted his graphical interpolation of the individual corrections to the Moon's mean longitude for intervals of a quarter of a century. I then throughout gave each equation of condition the weight of unity, as I look with suspicion on assigning different weights to the observations. I took out from the theory all of Hansen's terms of long period due to the action of the planets on the Moon, and substituted an inequality of long period, which goes through precisely the same changes in 2831 years, and is made up of the two composite periods of  $257\frac{4}{11}$  years and 149 years respectively. This inequality depends upon the distance of the Moon from the plane of the Sun's equator, when the Moon arrives at conjunction with the Sun. The following is the correction to the Moon's mean longitude which I have determined:—

$$\delta\epsilon = \delta v + \delta nz.$$

In which

$$\delta v = +0''.23 - 29''.35 T - 3.976 T^2.$$

And

$$\delta nz = -(V_1 + V_2 + M) + (14''.1 \sin A + 0''.9 \sin A')$$

in which  $T$  is the number of centuries from the epoch of Hansen's Tables, viz.: January 0.0 1800;  $\delta v$  is the correction to the Moon's true longitude, and  $\delta nz$  to that of the fundamental argument. The terms  $V_1$ ,  $V_2$ , and  $M$  are the values assigned by Hansen in the respective tables, xlii., xli., and xl., with the arguments 31, 30, and 29. The outstanding residuals are as follows:—

1625	+ 2''.1	1725	- 1''.4	1825	- 0''.2
1650	- 2.4	1750	- 1.4	1850	+ 0.8
1675	- 0.4	1775	+ 1.0	1875	- 0.5
1700	+ 0.1	1800	+ 0.7		

I have calculated a table at five-years intervals between A D 1600 and A D 2000 inclusive; of the values of  $\delta\epsilon$ , which represents the correction to the Moon's mean longitude; and it is a well-known fact that Hansen, being unable to determine the coefficients of his *Venus* terms, from theory, assigned such values to them as brought observation and theory into agreement during the period 1750-1850, but this he only attained by sacrificing accuracy before and after this interval. Now, if my corrections to the Moon's mean longitude are well founded, they ought to

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